Electromagnetic-wave propagation and amplification in overdense plasmas: Application to free electron lasers

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Considering a propagation of electromagnetic (EM) waves through overdense homogeneous plasmas, a new regime of amplification of EM waves with frequencies below the plasma frequency has been found. An application to free electron lasers is suggested. [S1063-651X(98)07912-4]

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I. INTRODUCTION

A large-amplitude electromagnetic (EM) wave propagating through plasmas is known to be subject to different nonlinear effects, such as Raman scattering, modulational instability, and self-focusing [1–3]. These nonlinear effects have great importance for fusion physics, laser-plasma acceleration, and EM-field harmonic generation. In particular, Compton and Raman scattering are intensively utilized in the concept of free electron lasers (FEL).

Usually, EM-wave–plasma interactions are investigated in underdense plasmas. It is interesting to consider the behavior of overdense plasmas in the presence of the EM fields, because it can lead to the increasing of nonlinear effects. The basis of such expectations is an enhancement of an amplitude of nonlinear plasma oscillations, driven by a ponderomotive force, with the increasing of plasma density. It seems that the main obstacle to utilizing the plasma oscillations is the impossibility for weak EM waves to propagate through plasmas if their frequencies are less than the Langmuir plasma frequency [1]. However, the presence of an EM field drastically changes the properties of plasmas. For example, it has been predicted theoretically and demonstrated experimentally [5–7] that an overdense plasma can be made transparent due to the hole-burning effect and due to the effect of increasing of the relativistic mass of the particles. These effects need incredibly high intensity of the EM field, and consequently, the realization of these effects looks possible only in the pulsed regime.

The coherent effects in the atoms are known to give rise to new phenomena such as quenching of spontaneous emission, electromagnetic induced transparency (EIT), lasing without population inversion, and high index of refraction without absorption [8]. For example, the EIT allows the propagation of EM waves without absorption in the medium which possesses losses for a single weak field.

Recently it has been shown that there are several types of coherent effects in plasmas which are similar to coherent effects in atomic media. The EIT in cold overdense plasmas [9] and a new concept of FEL [10] has been proposed. These effects are nonrelativistic and need a moderate level of EM-wave intensities.

In this paper we consider an EM-wave propagation in homogeneous, overdense plasmas. For the sake of simplicity we limit ourselves to a one-dimensional model, and take into account interaction with electrons only. Developing the ideas of coherence effects in plasmas, we show a possibility of EM-wave propagation and amplification in overdense plasmas and propose application of the effects to the FEL. These effects result from the modulation of the plasma density by coherent electromagnetic fields.

We consider the strict dispersion relation for the EM waves in plasmas in the presence of the strong EM field [11,12], including both the Stokes and anti-Stokes waves, and investigate the stability of an overdense plasma – EM-wave compound. We show the dependence of the system stability on the drive intensity and find the threshold behavior of the plasma instability. The plasma has only a relativistic modulational instability (RMI) for a sufficiently low drive intensity. In this region we find the EIT effect and show that the EIT gap is much narrower than has been predicted in [9]. We show that there exist unstable Stokes EM waves with frequencies smaller than the plasma frequency when the drive intensity is above the instability threshold. The differences between these instabilities and underdense plasma instabilities are found and analyzed. The dependence of the instabilities and the EIT on the temperature in warm overdense plasmas is discussed. Finally, we apply our analysis for the intense electron beams and using the Lorentz transformation investigate dispersion relations for a FEL-like system. We find a new regime of amplification for such a system.

II. EQUATIONS FOR WAVES IN PLASMAS

An interaction of EM waves with a cold, homogeneous, collisionless plasma is described by the set of self-consistent equations [11], which consists of the equations of motion of a single charged particle in the EM field, the continuity equation, and the wave equations for fields.

The equation of continuity and the wave equations for the fields have the forms

$$e\frac{\partial n}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{J}=0, \tag{1}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\mathbf{A}=\frac{4\pi}{c}\mathbf{J}, \tag{2}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\phi=4\pi en, \tag{3}$$

where e is an electron charge, c is the speed of light, $\mathbf{J}$ is the density of the electron current

$$\mathbf{J}=(n_0+n)e\mathbf{v},$$

$\mathbf{v}=(v_x,v_y,v_z)$ is an electron velocity, n_0 is the density of the plasma, n is the electron density of longitudinal plasma oscillations, ϕ is the electric potential connected with these plasma oscillations, and $\mathbf{A}$ is a vector potential of EM field. These potentials obey the Lorentz gauge condition

$$\frac{1}{c}\frac{\partial\phi}{\partial t}+\nabla\cdot\mathbf{A}=0. \tag{4}$$

Classical dynamics of electrons in the external fields can be described by the Hamiltonian

$$H=\gamma mc^2=c\sqrt{\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+m^2c^2}+e\phi,$$

where γ is referred to as a Lorentz factor,

$$\gamma=\left(1-\frac{\mathbf{v}^2}{c^2}\right)^{-1/2},$$

m is an electron mass, and $\mathbf{p}$ is a canonical momentum of electron.

To proceed further we choose $\mathbf{A}$ as $(0,A_y,A_z)$. The set of equations of motion for an electron in the external fields has the form

$$\dot{x}=\frac{\partial G}{\partial p_x}=\frac{p_x}{m\gamma},\quad \dot{y}=\frac{\partial H}{\partial p_y}=\frac{p_y-eA_y/c}{m\gamma},$$

$$\dot{z}=\frac{\partial H}{\partial p_z}=\frac{p_z-eA_z/c}{m\gamma},\quad \dot{p}_x=-\frac{\partial H}{\partial x}=0,\quad \dot{p}_y=-\frac{\partial H}{\partial y}=0, \tag{5}$$

$$\dot{p}_z=-\frac{\partial H}{\partial z}=\frac{p_z-eA_z/c}{m\gamma}\frac{e}{c}\frac{\partial A_z}{\partial z}+\frac{P_y-eA_y/c}{m\gamma}\frac{e}{c}\frac{\partial A_y}{\partial z}-e\frac{\partial\phi}{\partial z},$$

$$\dot{\gamma}mc^2=-ev_z\left(\frac{\partial\phi}{\partial z}+\frac{1}{c}\frac{\partial A_x}{\partial t}\right)-ev_y\frac{1}{c}\frac{\partial A_y}{\partial t}.$$

Using the equations of motion (5) together with the equation of continuity (1), and assuming the small relativistic change of the electron mass due to interaction with the EM waves, we obtain the equation for the plasma oscillations

$$\frac{d^2n}{dt^2}+\frac{\omega_p^2}{\gamma_z^3}n=\frac{e^2n_0}{2m^2c^2\gamma_z^2}\frac{\partial}{\partial z}\left(\frac{v_z^0}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)A_y^2, \tag{6}$$

where v_z^0 is the mean longitudinal velocity of plasma relative to the laboratory frame of reference

$$\gamma_z=\left[1-\left(\frac{v_z^0}{c}\right)^2\right]^{-1/2}.$$

The wave equation for EM fields (2) can be rewritten as

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{\omega_p^2}{\gamma_z c^2}\right)A_y =\frac{\omega_p^2}{c^2\gamma_z}\left(-\frac{n}{n_0}+\frac{e^2A_y^2}{2m^2c^4}+\gamma_z^2\frac{v_z^0(v_z-v_z^0)}{c^2}\right)A_y, \tag{7}$$

where $\omega_p^2=4\pi n_0e^2/m$ is the plasma frequency.

The relation between the electron density n and the longitudinal velocity v_z can be obtained from the modified continuity equation

$$\dot{n}=-n_0\frac{\partial}{\partial z}(v_z-v_z^0). \tag{8}$$

Equations (6)–(8) form the complete set describing the EM-wave–plasma interaction. Obtained from relativistic invariant equations (1)–(5), the set is relevant in any frame of reference. For example, it can be used for investigation of EM-wave scattering by plasmas or by electron beams.

In the case of underdense plasmas, Eqs. (6) and (7) can be simplified by neglecting the terms, which are proportional to the plasma frequency, in the left sides of the equations. This simplification corresponds to a consideration of an interaction between EM waves via a single electron. The interaction between the electrons is neglected (this is the so-called Compton regime). For the dense and overdense plasmas the collective effects play an important role and such a simplification is no longer valid.

III. DISPERSION RELATION FOR PLASMAS

To analyze a possibility of wave amplification we make a harmonic analysis of Eqs. (6)–(8) and obtain the dispersion relation for plasmas. Such an analysis has been provided in a large number of papers (for example, see [11–13]), and we do not repeat it here.

Let us use the vector potential of EM waves in plasmas in the form

$$\begin{aligned}\mathbf{A}(\mathbf{z},t)=\{&\mathbf{A}_0+\mathbf{A}_a\exp[i(\mathbf{k}\cdot\mathbf{z}-\omega t)]\\&+\mathbf{A}_S\exp[-i(\mathbf{k}^*\cdot\mathbf{z}-\omega^* t)]\}\\&\times\exp[i(\mathbf{k_0}\cdot\mathbf{z}-\omega_0 t)]+\text{c.c.}\end{aligned} \tag{9}$$

We assume that the sideband amplitudes $\mathbf{A}_a$ (anti-Stokes) and $\mathbf{A}_S$ (Stokes) are much smaller compared with carrier-wave amplitude $\mathbf{A}_0$, and that $\mathbf{A}_S$, $\mathbf{A}_a$, and $\mathbf{A}_0$ are linearly polarized, parallel to each other, and perpendicular to the carrier-wave vector.

For the Stokes and anti-Stokes sideband amplitudes the next set of equations can be obtained [11]:

$$D_+A_a+\tilde{\omega}_p^2\Gamma_0(1-c^2\tilde{k}^2/D)(A_a+A_S^*)=0, \tag{10}$$

$$D_- A_S^* + \tilde{\omega}_p^2 \Gamma_0 (1 - c^2 \tilde{k}^2 / D)(A_a + A_S^*) = 0, \tag{11}$$

where $\Gamma_0 = e^2 |\mathbf{A}_0|^2 / m^2 c^4$ is the nonlinear coupling coefficient,

$$D(\tilde{\omega}, \tilde{\mathbf{k}}) = \tilde{\omega}^2 - \tilde{\omega}_p^2$$

is the Langmuir-wave dispersion function,

$$D_\pm(\tilde{\omega}, \tilde{\mathbf{k}}) = \tilde{\omega}^2 \pm 2(\tilde{\omega}_0 \tilde{\omega} - c^2 \tilde{\mathbf{k}} \cdot \tilde{\mathbf{k}}_0) - c^2 \tilde{k}^2$$

are the light-wave dispersion functions, and $\tilde{\omega}_p, \tilde{k}$, and $\tilde{\omega}$ are the plasma frequency, wave vector, and frequency of the plasma wave in the moving frame of reference where the plasma is at rest,

$$\tilde{\omega}_p^2 = \frac{\omega_p^2}{\gamma_z}, \quad \tilde{k} = \left(k - \frac{v_z^0 \omega}{c^2} \right) \gamma_z, \quad \tilde{\omega} = (\omega - v_z^0 k) \gamma_z.$$

From Eqs. (10) and (11) we obtain the dispersion relation for plasma oscillations [11,12]:

$$\mathcal{D}(\tilde{\omega}, \tilde{k}) = D_+ D_- + \tilde{\omega}_p^2 \Gamma_0 (1 - c^2 \tilde{k}^2 / D)(D_+ + D_-) = 0. \tag{12}$$

It should be mentioned here that the dispersion relation (12) is valid independent of the frame of reference. This result is rather obvious because Eq. (12) is based on the equations which are invariant relative to the Lorentz transformations.

We consider a rather weak interaction plasma field ($\Gamma_0 \ll 1$), so the carrier-wave dispersion relation has the form

$$\tilde{\omega}_0^2 = \tilde{\omega}_p^2 (1 - \Gamma_0) + c^2 \tilde{k}_0^2. \tag{13}$$

In this case the unstable solutions of Eqs. (6)–(8) automatically mean the instability of sideband waves. Analyzing the roots of Eq. (12) we investigate amplification and generation of the Stokes and anti-Stokes waves in plasmas.

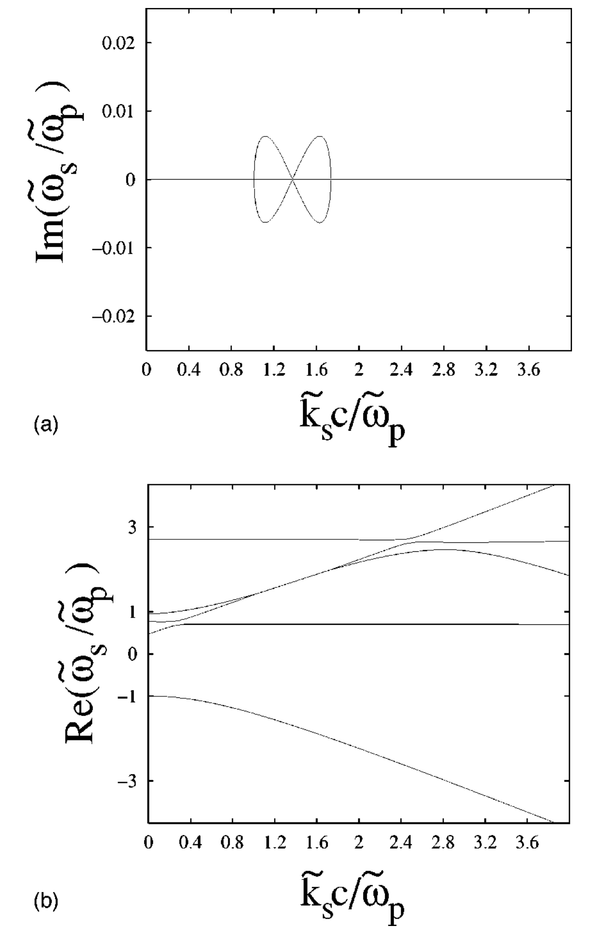

FIG. 1. (a) The temporal growth rate of the Stokes EM wave (the relativistic modulational instability) Im $\tilde{\omega}_S/\tilde{\omega}_p$ as a function of the real longitudinal wave vector $\tilde{k}_S c/\tilde{\omega}_p$, obtained for the drive EM field with the frequency $\tilde{\omega}_0 = 1.7\tilde{\omega}_p$ and the coupling constant $\Gamma_0 = 0.02$. (b) The dispersion of the Stokes EM wave Re$\tilde{\omega}_S/\tilde{\omega}_p$, corresponding to (a), as a function of the real longitudinal wave vector $\tilde{k}_S c/\tilde{\omega}_p$.

IV. ANALYSIS OF STABILITY IN COLD PLASMAS

To consider an amplification or an attenuation of EM waves propagating through plasmas we analyze the dispersion relation (12) obtained in the preceding section. It is instructive to remember that there are two approaches to analyze the propagation of the EM wave in plasmas. Let us suppose that the initial distribution of the field in the interaction region is known, and the task is to find the time-dependent behavior of the EM field. In this case the dispersion equation should be solved with respect to the frequency regarding the wave vector to be real. An appropriate imaginary part of complex frequency, that is a root of Eq. (12), indicates that there is an instability of the plasma wave, and EM waves with the real part of this frequency can be generated by the system. Temporal consideration is useful in the case of infinite uniform plasmas or finite plasmas with periodic boundary conditions.

In the other case, when the EM fields on the plasma-vacuum boundary are known, the task is finding the propagation of the EM waves through plasmas using these boundary conditions. Solving the dispersion equation with respect to the wave vector for the real frequencies gives us an imaginary part of the wave vector which corresponds to the wave attenuation or amplification. This approach is valid for the convective instabilities [2]. For absolute instabilities the problem is more complicated. There is a situation when there exist complex frequencies for real wave vectors (temporal picture) while for real frequencies we have only real wave vectors [4]. To find an amplification in this case one should know the geometry of the system.

We numerically solve the dispersion equation (12), which is fourth order of the wave vector and sixth order of the frequency $\tilde{\omega}$, and investigate the dependence of the plasma instabilities on the driving field intensity. For simplicity, we limit ourselves to copropagation and contrapropagation ($\tilde{\mathbf{k}} \| \tilde{\mathbf{k}}_0$) of the carrier and sideband waves.

In Figs. 1–4, we depict the solutions of Eq. (12) for the monochromatic driving field with the frequency $\tilde{\omega}_0 = 1.7\tilde{\omega}_p$. It is clear that for such a driving field the Stokes sideband has frequency smaller than plasma frequency $\tilde{\omega}_p$.

To begin with, we demonstrate that the low intensity of the driving field (for example, we set the parameter $\Gamma_0 = 0.02$) there exists only an instability due to a near-resonant

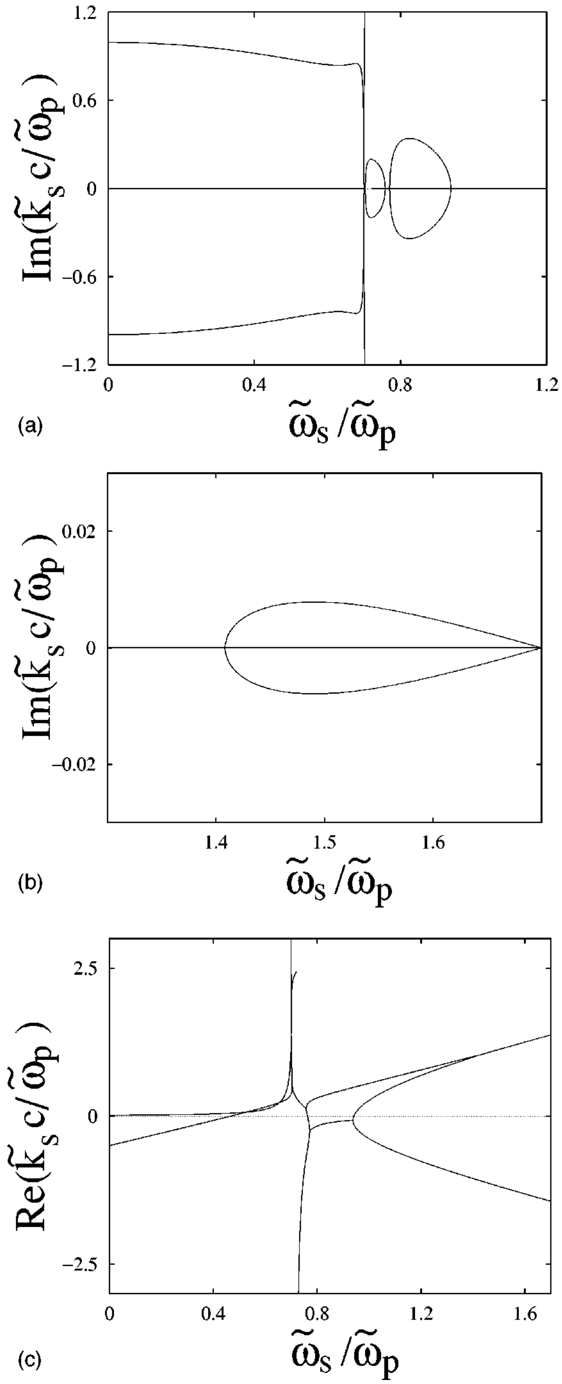

FIG. 2. (a) The imaginary part of the wave vector Im $\tilde{k}_S c/\tilde{\omega}_p$ as a function of the real frequency of the Stokes wave $\tilde{\omega}_S/\tilde{\omega}_p$. The relativistic modulational instability (b) corresponds to the branch close to $\tilde{\omega}_S=\tilde{\omega}_0$. The other branches characterize the evanescent field. The EIT gap is clearly seen at the vicinity of the plasma resonance. (c) The dispersion of the Stokes EM wave, corresponding to (a), (b), Re $\tilde{k}_S c/\tilde{\omega}_p$ as a function of the real frequency of the Stokes EM wave $\tilde{\omega}_S/\tilde{\omega}_p$.

interaction of a carrier wave and its Stokes and anti-Stokes sidebands [11], a so-called relativistic modulational instability in plasmas. In Fig. 1(a) we depict the temporal growth rate of the plasma wave (and, consequently, the sideband fields) as a function of the real wave vector. The real part of the plasma oscillation frequencies is shown in Fig. 1(b). The

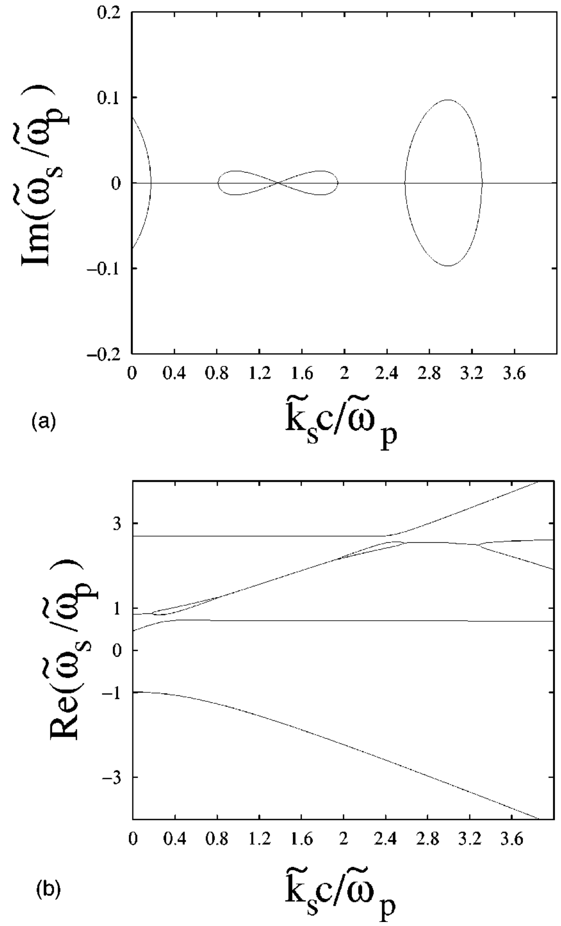

FIG. 3. (a) The temporal growth rate of the Stokes EM wave Im $\tilde{\omega}/\tilde{\omega}_p$ as a function of the real longitudinal wave vector $\tilde{k}_S c/\tilde{\omega}_p$, obtained for the drive EM field with the frequency $\tilde{\omega}_0=1.7\tilde{\omega}_p$ and the coupling constant $\Gamma_0=0.04$, which lies above the threshold. (b) The dispersion of the Stokes wave Re $\tilde{\omega}/\tilde{\omega}_p$, corresponding to (a), as a function of the real longitudinal wave vector $\tilde{k}_S c/\tilde{\omega}_p$.

RMI leads to growth of forward scattered Stokes and anti-Stokes waves because the corresponding wave vectors $\tilde{k}_a=\tilde{k}_0+\tilde{k}$ and $\tilde{k}_S=\tilde{k}_0-\tilde{k}$ have the same signs. As usually occurs for the RMI, $|\tilde{k}|$ is much less than $|\tilde{k}_0|$. It is clearly seen in Fig. 1(b) that there is no Raman instability in the overdense plasma (see, for example, [14]) for such a driving field.

Let us now consider the solution of the dispersion relation with respect to wave vectors $\tilde{k}$ keeping the real frequencies $\tilde{\omega}$. Imaginary and real parts for $\tilde{k}$ are shown in Fig. 2. Close to the point $\tilde{\omega}=0$ one finds the branch of spatial instability [Fig. 2(a)], which corresponds to the temporal RMI growth rate [Fig. 1(a)]. The real part of the wave vector corresponding to the spatial instability is depicted in Fig. 2(b). A comparison of Fig. 1 with Fig. 2 shows that, for the case of the RMI, it is possible to use either a temporal or a spatial picture. The results of such considerations are the same independent of the manner of investigation, because the RMI is convective.

It is easily seen in Fig. 2(a) that there are other complex roots $\tilde{k}$ of Eq. (12). These wave vectors do not correspond to the RMI. Therefore one can conclude that these branches

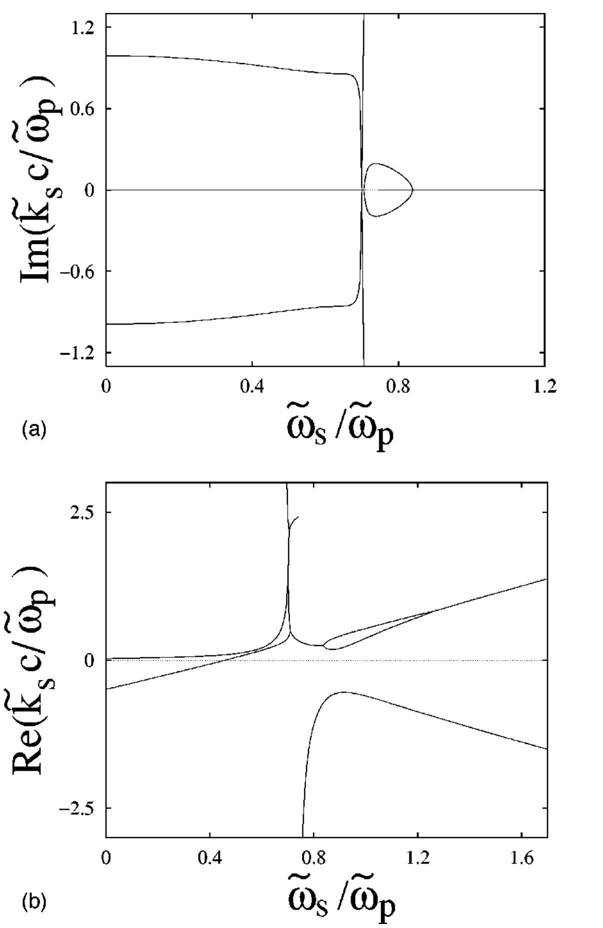

FIG. 4. (a) The imaginary part of the wave vector $\mathrm{Im}\,\tilde{k}_S c/\tilde{\omega}_p$ as a function of the real frequency of the Stokes EM wave $\tilde{\omega}_S/\tilde{\omega}_p$. The relativistic modulational instability corresponds to the branch close to $\tilde{\omega}_S=\tilde{\omega}_0$. The other branches correspond to the evanescent field. There is no branch which corresponds to the temporal plasma instability (a). (b) The dispersion of the Stokes EM wave, corresponding to (a), $\mathrm{Re}\,\tilde{k}_S c/\tilde{\omega}_p$ as a function of the real frequency of the Stokes EM wave $\tilde{\omega}_S/\tilde{\omega}_p$.

correspond to evanescent waves [4] and cannot be useful to amplification. The EM waves with frequencies corresponding to such $\tilde{k}$ are usually reflected by the plasma. However, one can see in Fig. 2(a), that some low frequency Stokes sidebands $(|\tilde{\omega}_S|=|\tilde{\omega}_0-\tilde{\omega}|<\tilde{\omega}_p)$ may propagate through plasmas without reflection or attenuation (so-called electromagnetically induced transparency, see [9]) in the presence of a driving field. We have revisited this effect and found that the EIT window in the plasmas looks different from that obtained earlier. This difference appears due to the Stokes and anti-Stokes wave interaction, which has been omitted in [9]. Consideration restricted by three wave interaction (the Stokes wave, the plasma wave, and the drive wave) works well for describing Raman scatterings, but in our case it leads to incomplete results. Really, if we consider only three wave interaction (this means that in the dispersion relation we should set $D_+ \gg D_-$ and therefore omit D_- where it is possible), we obtain the wide EIT gap in plasmas. The width of this gap is about to $3\,\Gamma_0\tilde{\omega}_p$ [9]. Taking the anti-Stokes sideband into account, one can make the inference that the EIT exists in cold plasmas, but the width of the gap is narrower [see Fig. 2(a)]. Moreover, taking into account the anti-Stokes wave leads to the substantial changing of the opposite to the EIT effect, predicted in [9]. There is a region near the anti-Stokes frequency (well above the plasma frequency, usually it is a transparent region in the linear regime) where the probe field is reflected from the plasmas. The physical cause of this effect is the same as for the EIT. A high frequency field reflects from the plasma due to a dynamical diffractional lattice, which is formed by the ponderomotive force, which acts on the charged particles in the presence of the EM fields. We found that this region is much wider compared with the result obtained in [9].

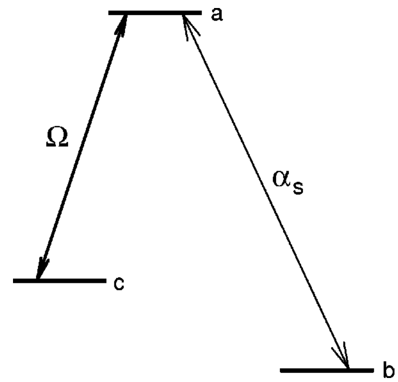

FIG. 5. The scheme of three-level atom in Λ configuration. Ω and α_S are the Rabi frequencies of the driving and probe coherent fields, respectively. The dispersion relation (16) is obtained for the probe field having the wave vector k_S and frequency ω_S.

Let us consider now the situation of a larger drive than in the previous case. For the intensity of the driving EM field being higher than some threshold value we get the temporal instability (the parameter $\Gamma_0=0.04$, see Fig. 3). This threshold value depends on the plasma and drive frequencies. This threshold value can be easily estimated in the limit of $D_+ \gg D_-$. The dispersion relation (12) in this case can be rewritten as

$$c^2\tilde{k}_S^2-\tilde{\omega}_S^2+\tilde{\omega}_p^2-\Gamma_0\tilde{\omega}_p^2\left(1+\frac{c^2(\tilde{k}_0-\tilde{k}_S)^2}{\tilde{\omega}_p^2-(\tilde{\omega}_0-\tilde{\omega}_S)^2}\right)=0, \tag{14}$$

and the threshold nonlinearity of interaction is determined by the value

$$\Gamma_0^{\mathrm{th}} \simeq \frac{[\tilde{\omega}_p^2-(\tilde{\omega}_0-\tilde{\omega}_S)^2](\tilde{\omega}_p^2-\tilde{\omega}_S^2)}{\tilde{\omega}_p^2(\tilde{\omega}_0^2-\tilde{\omega}_S^2)}. \tag{15}$$

Therefore for the drive EM field with the intensity higher than the threshold $(\Gamma_0>\Gamma_0^{\mathrm{th}})$ there exists a plasma instability. It should be mentioned here that there is no corresponding spatial instability (see in Fig. 4) for this temporarily unstable branch.

Let us note here that the similarity between plasmas and driven three-level systems can be established on the base of the simplified dispersion relation (14). In the case of three-level atoms driven by a coherent field the dispersion relation consists of two parts (to be certain we refer to the Λ system, see Fig. 5) [8]:

$$\left(ik_S - i\frac{\omega_S}{c} - \xi\frac{n_a - n_b}{\Gamma_{ab}} \right) - |\Omega|^2 \xi \frac{n_c - n_a}{\Gamma_{ab}\Gamma_{ca}\Gamma_{cb}} = 0, \quad (16)$$

where n_a, n_b, and n_c are the populations of the levels a, b, and c; $\Gamma_{ij} = \gamma_{ij} + i\Delta_{ij}$, γ_{ij} is the relaxation of coherence between levels i and j, Δ_{ij} is the detuning for the transition between levels i and j; ξ is a coupling constant between a radiation and a matter which is equal to $(3/8\pi)\lambda^2 N$ for the case of the radiative broadening transition, and N is the density of atoms. The first part in parentheses is the linear absorption and dispersion which exists without the driving field as well as in the case of the plasma (14) the term without Γ_0 is a linear dispersion of the EM waves in plasmas. The second part is the driving term. It is this term that leads to the electromagnetically induced transparency and the lasing without inversion (LWI) [8]. The first one allows the propagation without absorption of the two EM waves which have the frequency difference being equal to the splitting of the ground levels. The second one leads, under certain conditions (for example, the driving field intensity should exceed some threshold value), to the amplification of one of the EM waves without population inversion. In the case of plasmas the role of the second, driving term, is similar. The plasma oscillations play the role of the low frequency coherence in the case of the Λ atoms. As has been shown in [9] the driving term leads to the EIT in plasmas, and, as we have just shown, to the instability of the plasma. Therefore the collective plasma effects are similar to the atomic coherent effects.

It is important to determine the nature of the instability in this case. According to the usual criterion [2,14], we solve numerically the set of equations

$$\mathcal{D}(\tilde{\omega}, \tilde{k}) = 0$$

$$\frac{\partial \mathcal{D}(\tilde{\omega}, \tilde{k})}{\partial \tilde{k}} = 0,$$

and find a complete set of roots. There are roots of the set of equations which correspond to unstable plasma behavior $\tilde{\omega} \simeq (0.84 + 0.09i)\tilde{\omega}_p$. We have an absolute type of a plasma instability. This instability is not a Raman one, because the frequency of the Stokes sideband lies below $\tilde{\omega}_p$. However, this instability can be used for EM-wave amplification and generation.

V. ANALYSIS OF STABILITY IN WARM PLASMAS

To answer the question of whether an experimental implementation of the results obtained in the preceding section is possible, we have to reconsider the analysis of stability for the case of warm plasmas and determine the role of thermal electron motion.

For the most interesting case of sufficiently cold plasmas, when $T \geqslant 0$ but still $\tilde{\omega} \gg \tilde{k} v_T$, where v_T is the thermal electron velocity, kinetic theory analysis shows that plasma frequency should be changed to $\tilde{\omega}_{pT} = \tilde{\omega}_L - i\gamma_L$, where

$$\tilde{\omega}_L^2 = \tilde{\omega}_p^2 + 3\tilde{k}^2 v_T^2, \qquad \gamma_L = \sqrt{\frac{\pi}{8}} \frac{\tilde{\omega}_p^2 \tilde{\omega}_L^2}{\tilde{k}^3 v_T^3} \exp\left(-\frac{\tilde{\omega}_L^2}{2\tilde{k}^2 v_T^2} \right).$$

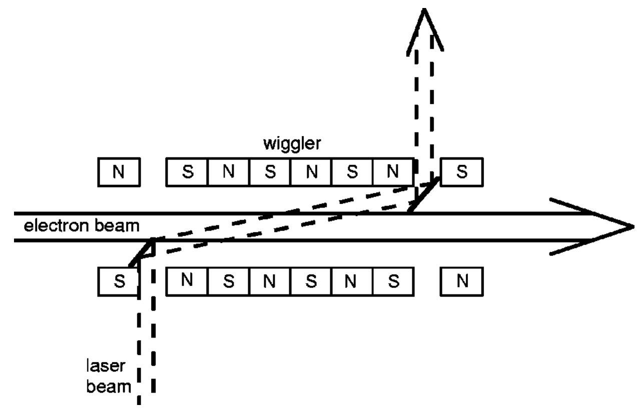

FIG. 6. The FEL-type setup for the demonstration of the amplification effect for the EM wave with the frequency below the plasma frequency.

Landau damping γ_L is negligible in our case due to small $\tilde{k}$. This modification changes the order of the dispersion relation for the wave vector $\tilde{k}$ and does not change the order for the frequency $\tilde{\omega}$.

The main changes of the wave properties between the warm and cold plasmas are as follows. The temporal picture and corresponding part of the spatial picture (the RMI branch for $\Gamma_0 = 0.02$) do not change much until v_T is less than $0.1c$, where c is the speed of light in vacuum, whereas the other branches of the spatial picture display significant changes for $v_T > 0.03c$.

The EIT gap demonstrates a threshold behavior. For the parameters chosen above, it disappears when the thermal electron velocity reaches the value $v_T \simeq 0.1c$.

The threshold for the temporal instability increases with the temperature, as is clearly seen from Eq. (15),

$$\Gamma_0(T) = \Gamma_0(0) + 3\tilde{k}^2 v_T^2 \left(1 - \frac{(\tilde{\omega}_0 - \tilde{\omega}_S)^2 \tilde{\omega}_S^2}{\tilde{\omega}_p^4} \right).$$

Increasing of the plasma temperature leads to vanishing of the nonlinear interaction between the sideband waves and the plasma. This is similar to destroying a coherence in atomic systems by the thermal motion. The phase and group velocities of sideband waves become very large. We would like to note that without nonlinearity these velocities and the wavelength of sideband waves go to infinity. This means that these waves cannot penetrate through plasmas.

VI. APPLICATION TO FREE ELECTRON LASERS

In conventional FELs [16], a linearly polarized static magnetic wiggler is used to generate stimulated radiation. In laser-pumped FELs, the static periodic magnetic wiggler is replaced by a counterstreaming intense laser field [17] (optical wiggler). It should be noted that for a relativistic beam the periodic magnetic field of the wiggler appears approximately, using Weiszacker-Williams approximation, as a plane EM wave [18]. Due to this fact, we neglect the difference between these types of wigglers for gain calculation.

We consider the setup presented in Fig. 6. The high density relativistic electron beam passes through the wiggler. In the frame of reference moving with the electron beam the wiggler can be considered as a strong driving field propagating through the plasma. It is similar to the configuration which we have analyzed above. The weak laser field to be amplified should be launched onto plasma in the presence of the strong pump field, as shown in Fig. 6. To calculate the increment of the instability for the weak laser field E_l propagating through the wiggler along the electron beam we use the transformed dispersion relation (12).

It is known that the moving electrons ''see'' the wiggler as almost transverse EM wave, the wave vector and frequency of which in the rest frame of the electron beam is determined as

$$\tilde{\omega}_0 = k_W v_z^0 \gamma_z, \quad \tilde{k}_0^2 = \left(\frac{k_W v_z^0 \gamma_z}{c} \right)^2 - \frac{\tilde{\omega}_p^2}{c^2},$$

where $k_W = 2\pi/\lambda_W$, λ_W is the wiggler wavelength, v_z^0 is the mean speed of the electron beam, and γ_z corresponds to the v_z^0 Lorentz factor. Therefore we can consider the wiggler field as the carrier wave and use for calculations the above results.

To determine the parameters of the amplified field in the laboratory frame we can use the Lorentz transformation. It is worthwhile to underline here that forward scattered waves can be backscattered relative to the electron beam in the laboratory frame under the condition $\tilde{\omega} > \tilde{k} v_z^0$. For the backscattered waves the frequency in the laboratory frame is higher $\omega = (\tilde{\omega} + \tilde{k} v_z^0)\gamma_z$. Thus, even for the Stokes sideband waves with the frequency smaller than the plasma frequency in the moving frame, in the laboratory frame the frequency is in the optical region for large γ_z.

Let us consider the situation when the wiggler field wavelength is $\lambda_W = 9$ cm, the wiggler field strength is $B_W = 2.7$ kG ($\Gamma_0 = 0.04$), the Lorentz factor is $\gamma_z = 40$, the longitudinal momentum spread is $\Delta\gamma = 10^{-3}\gamma_z$, and the electron density is $10^{14}\,\mathrm{cm}^{-3}$. Corresponding carrier-wave frequency in the moving frame is $\tilde{\omega}_0 = 8.38 \times 10^{11}\,\mathrm{s}^{-1}$, the plasma frequency is $\tilde{\omega}_p = 5 \times 10^{11}\,\mathrm{s}^{-1}$, the thermal velocity is $v_T = \Delta\gamma/\gamma_z c = 3 \times 10^7$ cm/s, and the corresponding temperature is 225.0 eV. For the probe laser with the frequency $\omega_f = 1.7 \times 10^{13}\,\mathrm{s}^{-1}$, which in the moving frame corresponds to the frequency $\tilde{\omega} = 4.2 \times 10^{11}\,\mathrm{s}^{-1}$, which is less than the plasma frequency, the increment of the instability is equal to $\mathrm{Im}(\tilde{\omega}) = 0.09\tilde{\omega}_p$.

The above estimation gives us the gain for the optical fields. The limitation of the gain in the optical region comes from the restriction of the plasma density in the electron beams, and this limits the applications of the concept under consideration into the generation of optical (or shorter wavelength) coherent fields. However, the FEL, based on the proposed effect, looks quite competitive with the Raman FELs, also utilizing the collective motion of plasma, to produce radiation in IR and far-IR regions [3].

In conclusion, we consider the dispersion relation of overdense homogeneous plasmas, and find a new regime of amplification EM waves with the frequencies below the plasma frequency. This regime does not involve Raman instability, because the pump frequency is lower than the double plasma frequency (according to [1]) and can be useful for improving free electron laser operation, because it broadens the boundaries of FEL operation to the low frequency region. Such devices have larger gain compared with the Compton and Raman FELs for the same amplitudes of the driving field. These devices can be used, for example, for producing infrared radiation with significantly higher brilliance than usual laboratory sources such as a mercury lamp.

We have found that the EIT window in the plasmas looks different from that obtained in [9]. The width of the gap is narrower [see Fig. 2(a)]. We have shown that a region near the anti-Stokes frequency (well above the plasma frequency, it is usually a transparent region in the linear regime) where the probe field is reflected from plasmas, is much wider than the region predicted in [9].

We should mention here that direct application of the results obtained above for plasmas is to be corrected, because of the inhomogeneity of real plasmas which should be taken into account. This inhomogeneity is the source of a set of problems. For example, it breaks the phase matching conditions for the waves in plasmas, which leads to a boundary reflection of the EM waves in the case of overdense plasmas. The regions with low plasma density experienced all types of EM-wave scattering, which can significantly diminish the intensity of the carrier wave and scatter the low frequency probe wave before they penetrate the plasma.

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APPENDIX A

To derive Eq. (7) we assume a small change of the electron mass due to interaction with the EM waves. This leads to the approximation

$$\gamma^{-1} = \left(1 - \frac{v_y^2 + v_z^2}{c^2} \right)^{1/2} \simeq \gamma_z^{-1} \left(1 - \gamma_z^2 \frac{v_y^2 + 2v_z^0(v_z - v_z^0)}{2c^2} \right), \tag{A1}$$

where v_z^0 is the velocity of the electron beam. For transversal velocity of the electron v_y we have equations of motion

$$\dot{y} = \frac{\partial H}{\partial p_y} = \frac{p_y - eA_y/c}{m\gamma}, \quad \dot{p}_y = -\frac{\partial H}{\partial y} = 0. \tag{A2}$$

Setting the initial transversal momentum of the electron to be equal to zero, $p_y = 0$, we obtain from Eq. (A2) the expression

$$\dot{y} = -\frac{eA_y}{mc\gamma}. \tag{A3}$$

Combining Eq. (A1) with Eq. (A3) and omitting the terms with higher order than $(eA_y/mc\gamma_z)^3$, we can express the transversal velocity of the electron in the form

$$v_y=-\frac{eA_y}{mc\gamma_z}\left(1-\gamma_z^2\frac{(eA_y/mc\gamma_z)^2+2v_z^0(v_z-v_z^0)}{2c^2}\right). \tag{A4}$$

We plug expression (A4) into the wave equation for A_y, Eq. (2),

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)A_y=\frac{4\pi}{c}(n_0+n)ev_y. \tag{A5}$$

Then, using the definition of the plasma frequency $\omega_p^2=4\pi e^2 n_0/m$ and expression (A4), we rewrite Eq. (A5) in the final form (7):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{\omega_p^2}{\gamma_z c^2}\right)A_y =\frac{\omega_p^2}{c^2\gamma_z}\left(-\frac{n}{n_0}+\frac{e^2A_y^2}{2m^2c^4}+\gamma_z^2\frac{v_z^0(v_z-v_z^0)}{c^2}\right)A_y. \tag{A6}$$

To derive an equation for the plasma oscillations (6) we use the equation of continuity (8)

$$\frac{\partial n}{\partial t}+\boldsymbol{\nabla}\cdot(n_0+n)\mathbf{v}=0.$$

We consider the one-dimensional case; the vector potential A_y depends on z and t only. Therefore, assuming small plasma perturbation $n\ll n_0$, we have

$$\nabla\cdot(n_0+n)\mathbf{v}=\frac{\partial}{\partial y}(n_0+n)v_y+\frac{\partial}{\partial z}(n_0+n)v_z =v_z\frac{\partial}{\partial z}n+n_0\frac{\partial}{\partial z}v_z.$$

In this case, the continuity equation has a form

$$\dot{n}=\frac{\partial n}{\partial t}+v_z^0\frac{\partial n}{\partial z}=-n_0\frac{\partial}{\partial z}v_z. \tag{A7}$$

We apply d/dt to Eq. (A7), and rewrite it as

$$\ddot{n}=-n_0\frac{\partial}{\partial z}\dot{v}_z. \tag{A8}$$

Using the equations of motion (5) we obtain the expression for a longitudinal electron velocity

$$m\gamma v_z=p_z-\frac{e}{c}\dot{A}_z, \tag{A9}$$

which gives us the convective derivative for v_z:

$$\dot{v}_z=\frac{1}{\gamma m}\left(-mv_z\dot{\gamma}+\dot{p}_z-\frac{e}{c}\dot{A}_z\right). \tag{A10}$$

Using Eq. (A3) and substituting the expressions for $\dot{\gamma}$, $\dot{p}_z$ [see Eq. (5)] into Eq. (A10) we rewrite the last one as

$$\dot{v}_z=-\frac{e}{\gamma\gamma_z^2 m}\left(\frac{\partial\phi}{\partial z}+\frac{1}{c}\frac{\partial A_z}{\partial t}\right)-\frac{e^2A_y}{\gamma^2m^2c^2}\left(\frac{v_z}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)A_y. \tag{A11}$$

Then we apply $\partial/\partial z$ to (A11) and, using expression (A1), obtain

$$\frac{\partial}{\partial z}\dot{v}_z=-\frac{e}{\gamma_z^3 m}\left(\frac{\partial^2\phi}{\partial z^2}+\frac{1}{c}\frac{\partial^2 A_z}{\partial z\partial t}\right) -\frac{e^2}{\gamma_z^2m^2c^2}\frac{\partial}{\partial z}A_y\left(\frac{v_z}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)A_y. \tag{A12}$$

The Lorentz gauge condition (4) gives

$$\frac{1}{c}\frac{\partial^2 A_z}{\partial z\partial t}=-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}. \tag{A13}$$

Taking into account

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\phi=4\pi en, \tag{A14}$$

we plug Eq. (A13) into the equation of continuity (A8) and obtain the final form of the equation for plasma oscillations (6)

$$\frac{d^2n}{dt^2}+\frac{\omega_p^2}{\gamma_z^3}n=\frac{e^2n_0}{2m^2c^2\gamma_z^2}\frac{\partial}{\partial z}\left(\frac{v_z^0}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)A_y^2, \tag{A15}$$

which corresponds to the equation obtained in [15].

APPENDIX B

Here we consider the frame of reference connected with the plasma $v_z^0=0$. To get the dispersion equation (12) we represent the vector potential of EM waves in plasmas in the form

$$\mathbf{A}(\mathbf{z},t)=\{\mathbf{A}_0+\mathbf{A}_a\exp[i(\tilde{\mathbf{k}}\cdot\mathbf{z}-\tilde{\omega}t)]+\mathbf{A}_S \times\exp[-i(\tilde{\mathbf{k}}^*\cdot\mathbf{z}-\tilde{\omega}^*t)]\}\exp[i(\tilde{\mathbf{k}}_0\cdot\mathbf{z}-\tilde{\omega}_0t)]+\text{c.c.},$$

assuming that the sideband amplitudes $\mathbf{A}_a$ (anti-Stokes) and $\mathbf{A}_S$ (Stokes) are small in comparison with $\mathbf{A}_0$, and that $\mathbf{A}_S$, $\mathbf{A}_a$, and $\mathbf{A}_0$ are linearly polarized, parallel to each other, and perpendicular to the carrier wave vector $\tilde{\mathbf{k}}_0$. $\tilde{\omega}_0$ is the frequency of the carrier wave. We chose the plasma density in the form

$$n=n_0\exp[i(\tilde{\mathbf{k}}\cdot\mathbf{z}-\tilde{\omega}t)]+\text{c.c.},$$

where $\tilde{k}$ and $\tilde{\omega}$ are wave vector and frequency of plasma wave.

Using equation for plasma oscillations (6)

$$\frac{d^2n}{dt^2}+\tilde{\omega}_p^2 n=\frac{e^2n_0}{2m^2c^2}\frac{\partial^2}{\partial z^2}A_y^2,$$

together with wave equation (7)

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{\tilde{\omega}_p^2}{c^2}\right)A_y=\frac{\tilde{\omega}_p^2}{c^2}\left(-\frac{n}{n_0}+\frac{e^2A_y^2}{2m^2c^4}\right)A_y,$$

we get for plasma oscillations

$$Dn=-\frac{e^2n_0}{2m^2c^2}\tilde{k}^2(A_0A_S^*+A_aA_0^*),$$

where $D=\tilde{\omega}_p^2-\tilde{\omega}^2$.

For fields we get

$$D_+A_a=\frac{\tilde{\omega}_p^2}{c^2}\left(-nA_0+\frac{e^2}{2m^2c^4}(|A_0|^2A_a+A_0^2A_S^*)\right),$$

$$D_-A_S^*=\frac{\tilde{\omega}_p^2}{c^2}\left(-n^*A_0+\frac{e^2}{2m^2c^4}[|A_0|^2A_S^*+(A_0^*)^2A_a]\right),$$

where $D_\pm=\tilde{\omega}^2\pm 2(\tilde{\omega}_0\tilde{\omega}-c^2\tilde{k}\tilde{k}_0)-c^2\tilde{k}^2$.

Finally we come to relations

$$D_+A_a=\frac{\tilde{\omega}_p^2}{c^2}\frac{e^2}{m^2c^4}(1-c^2\tilde{k}^2/D)(|A_0|^2A_a+A_0^2A_S^*),$$

$$D_-A_S^*=\frac{\tilde{\omega}_p^2}{c^2}\frac{e^2}{m^2c^4}(1-c^2\tilde{k}^2/D)[|A_0|^2A_S+(A_0^*)^2A_a],$$

which gives us the dispersion relation (12).

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